Name: GRADERS: PINNER, ELSTON, HALLAUER, RICE

Recitation Instructor:

Recitation Day & Time:

Total/100	Grade

MATH 100 College Algebra – Exam III

7:15–8:15pm, Tuesday April 11, 2006

Check that you have all three pages. SHOW ALL YOUR WORK. Write complex answers in $a + bi$ form.

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1. (16 points) Perform the indicated divisions:

(a) $(x^3 + 8x^2 + 5x - 3) \div (x - 1)$. Quotient= $x^2+9x+14$, Remainder= 11.

$$
\begin{array}{r|l}
 & x^2 + 9x + 14 \\
x-1 & x^3 + 8x^2 + 5x - 3 \\
 & x^3 - x^2 \\
 & 9x^2 + 5x - 3 \\
 & 9x^2 - 9x \\
 & 14x - 3 \\
 & 14x - 14 \\
 & 11
\end{array}
$$

OR

$$
\begin{array}{r|rrrr}
 & 1 & 8 & 5 & -3 \\
1 & & 1 & 9 & 14 \\
\hline
 & 1 & 9 & 14 & 11
\end{array}
$$

(b) $(3x^4 - 1) \div (x^2 + x + 4)$. Quotient= $3x^2-3x-9$, Remainder= $21x+35$.

$$
\begin{array}{r|l}
 & 3x^2 - 3x - 9 \\
x^2+x+4 & 3x^4 + 0x^3 + 0x^2 + 0x - 1 \\
 & 3x^4 + 3x^3 + 12x^2 \\
 & -3x^3 - 12x^2 \qquad - 1 \\
 & -3x^3 - 3x^2 - 12x \\
 & -9x^2 + 12x - 1 \\
 & -9x^2 - 9x - 36 \\
 & 21x + 35
\end{array}
$$

2. (7 points) $f(x) = 6x^4 - 19x^3 + 29x^2 - 15x - 9$.

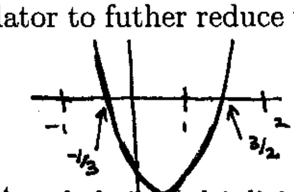
(a) Use the rational zeros test to list all the *possible* rational zeros of the polynomial f.

a/b a divides 9 $\Rightarrow a = \pm1, \pm3, \pm9$ $\Rightarrow a/b \in \{\pm1, \pm3, \pm9, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}\}$
b divides 6 $b = 1, 2, 3, 6$

(b) What are the *actual* rational zeros of f? You may use your calculator to futher reduce the possibilities you listed in (a).

$f(-1/3) = 0$

$f(3/2) = 0$



3. (6 points) Find the zeros of the polynomial $p(x) = x^6 + 2x^5 - 8x^4$ and their multiplicities.

$p(x) = x^4(x^2+2x-8)$
$= x^4(x+4)(x-2)$

Zeros: $x = 0$ multiplicity 4
$x = -4$ multiplicity 1
$x = 2$ multiplicity 1

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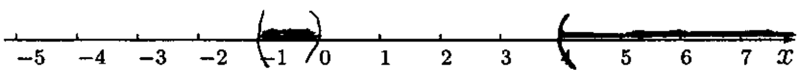
4. (7 points) Solve the polynomial inequality $6x^4 \leq 3x^3$. Give the solution set in interval notation. (If you use your calculator, explain what you are doing.)

For $x<0$ we have $6x^4>0$ and $3x^3<0$ so $6x^4>3x^3$. Thus our inequality has no negative solutions. For $x=0$, $6x^4=0$ and $3x^3=0$, so $x=0$ is a solution. For $x>0$ we have $3x^3>0$ so $6x^4 \leq 3x^3 \Leftrightarrow \frac{6x^4}{3x^3} \leq \frac{3x^3}{3x^3} \Leftrightarrow 2x \leq 1 \Leftrightarrow x \leq \frac{1}{2}$.

Thus the solution set is $[0, \frac{1}{2}]$.

5. (9 points) Solve the rational inequality $\frac{4}{x} < \frac{5}{x+1}$ and sketch the solution set on the real number line below. Remember to indicate the status of endpoints.



$\frac{4}{x} < \frac{5}{x+1}$ is equivalent to $0 < \frac{5}{x+1} - \frac{4}{x}$. Thus the related function is $f(x) = \frac{5}{x+1} - \frac{4}{x} = \frac{5x-4(x+1)}{x(x+1)} = \frac{x-4}{x(x+1)}$.

Note that $x=0$ and $x=-1$ are not allowed in our inequality. Moreover, $f(x)=0 \Leftrightarrow x=4$, so the critical points are $-1, 0, 4$.

Interval	$(-\infty, -1)$	$(-1, 0)$	$(0, 4)$	$(4, \infty)$
Test value	$f(-2) = -3$	$f(-\frac{1}{2}) = 18$	$f(1) = -1.5$	$f(5) \cong 0.03$
Sign of $f(x)$	$-$	$+$	$-$	$+$

so solution set for $0 < f(x)$ is $(-1, 0) \cup (4, \infty)$

6. (10 points) (a) If $f(x) = x^2 - x + 2$ and $g(x) = x - 3$, then the composition $(f \circ g)(x) =$ $x^2 - 7x + 14$,

$$(f \circ g)(x) = f(g(x)) = f(x-3) = (x-3)^2 - (x-3) + 2 = x^2 - 6x + 9 - x + 3 + 2$$
$$= x^2 - 7x + 4$$

(b) If $h(x) = 2x^3 - 7$, then the inverse function $h^{-1}(x) =$ $\sqrt[3]{\frac{x+7}{2}}$.

Write $y = 2x^3 - 7$. Interchange x and y to get $x = 2y^3 - 7$.
Solve for y: $\frac{x+7}{2} = y^3$ so $y = \sqrt[3]{\frac{x+7}{2}}$. Replace y by $h^{-1}(x)$.

7. (2 points) The graph of a polynomial f has 7 turning points. The smallest degree f can have is 8.

8. (5points) Express in terms of natural logarithms $\log_4 17 =$ $\frac{\ln 17}{\ln 4}$ (Use Change-of-base Formula)

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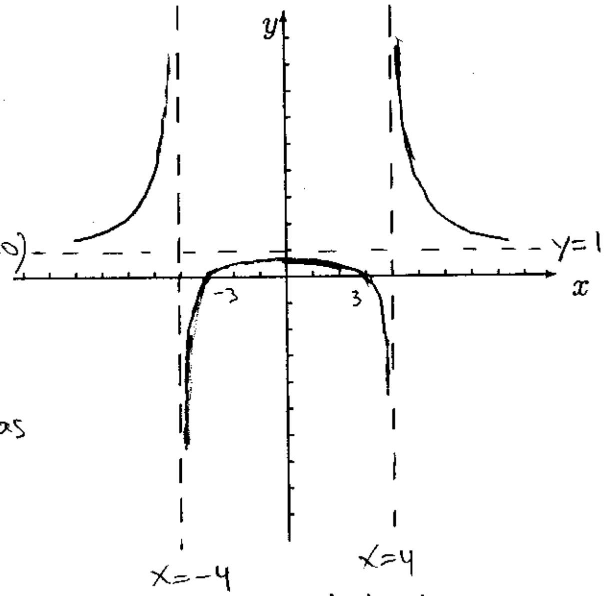
Note: A graph NEVER touches it's vertical asymptotes!

9. (18 points) Provide the following information and use it to graph the rational function

$$g(x) = \frac{x^2-9}{x^2-16}.$$



All asymptotes dotted

(a) The y-intercept: $g(0) = \frac{-9}{-16} = \frac{9}{16}$ so y-intercept is $(0, 9/16) = (0, 0.5625)$

(b) The x-intercept(s):

$x^2-9=(x-3)(x+3)=0 \iff x=\pm 3$ so x-intercepts are $(3,0)$ and $(-3,0)$

(c) Equations of any vertical asymptotes:

$x^2-16=(x-4)(x+4)=0 \iff \boxed{x=4,\quad x=-4}$

(Note $x^2-9\neq 0$ for $x=\pm 4$!)

(d) Equation of any horizontal asymptote:

Degree of x^2-9 = Degree of x^2-16, so g has horizontal asymptote $y=\frac{1}{1}=1$

(e) Symmetry: Is $g(x)$ even, odd or neither?

$g(-x) = \frac{(-x)^2-9}{(-x)^2-16} = \frac{x^2-9}{x^2-16} = g(x)$ so g is even

10. (12 points) For the following rational functions find the vertical, horizontal and oblique asymptotes (write an **equation** for the asymptote in the space provided or N/A if there is not an asymptote of that type):

	vertical	horizontal	oblique
$f(x)=\dfrac{x+3}{2x^2+1}$	$2x^2+1>0$ for all real x so N/A	Deg$(x+3)<$ Deg$(2x^2+1)$ so $\boxed{y=0}$	Deg$(x+3)\neq$ Deg$(2x^2+1)+1$ so N/A
$f(x)=\dfrac{x^2-7x}{x+4}$	$x+4=0$ and $x^2-7x\neq 0$ for $x=-4$ so $\boxed{x=-4}$	Deg$(x^2-7x)>$ Deg$(x+4)$ so N/A	Deg$(x^2-7x)=$ Deg$(x+4)+1$ and $\boxed{y=x-11}$

$$\begin{array}{r}
x-11 \\
x+4\,\overline{)\,x^2-7x+0} \\
x^2+4x \\
\hline
-11x+0 \\
-11x-44 \\
\hline
44
\end{array}$$

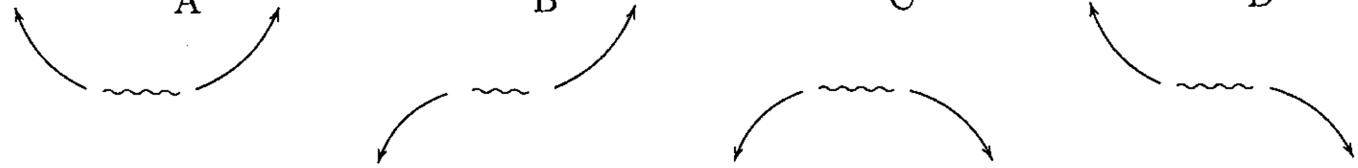
Above Deg$(x+3)$ means degree of polynomial $x+3$ etc.

10. (4 points) The polynomial $x^4-6x^3+33x^2-102x-26$ has zeros $1-5i$ and $2+\sqrt{5}$. The other zeros are $1+5i$, $2-\sqrt{5}$. (Hint: No calculation is required.)

See Section 3.3 p. 217 in Text Book.

12. (4 points) Use the leading-term test to determine which graph best represents the behavior of the given polynomial as $x\to\infty$ and $x\to-\infty$: (Use Leading-Term Test Sect. 3.1 p. 197.)

A B C D



(a) $-2x^6+5000x^4+3$. Circle One: A B C D.

Leading Term $=-2x^6$

Even degree Negative Leading Coef.

(b) $-3000x^2+x^3+700$. Circle One: A B C D.

Leading Term $=x^3=1x^3$

Odd degree, Positive Leading Coef.

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