Name:

Recitation Instructor:

Recitation Day & Time: Graders: Burckel, Elston, Hallauer, Rice

Total/100	Grade

MATH 100 College Algebra – Exam II

7:15–8:15pm, Tuesday March 7, 2006

Check that you have all three pages. SHOW ALL YOUR WORK. Write complex answers in $a + bi$ form.

1. (8 points) Write the complex numbers in the standard form $a + bi$, where a, b are real numbers:

(a) $\sqrt{-25} + i^{17} = \sqrt{25}\,i + i^{4\cdot 4+1} = 5i + (i^4)^4 i^1 = 5i + (1)^4 \cdot i = 6i = 0 + 6i$

(b) $\dfrac{2+i}{4-i} = \dfrac{2+i}{4-i} \cdot \dfrac{4+i}{4+i} = \dfrac{(2+i)(4+i)}{(4-i)(4+i)} = \dfrac{2\cdot 4 + 2i + i4 + i\cdot i}{4^2 - i^2} = \dfrac{8+6i-1}{16-(-1)} = \dfrac{7+6i}{17} = \dfrac{7}{17} + \dfrac{6}{17}i$

$(x-y)(x+y) = x^2 - y^2$

2. (7 points) State the Quadratic Formula (for the solutions of the general quadratic $ax^2 + bx + c = 0$) and use it to solve the equation $2x^2 + 10 - 8x = 0$. Specify your values of a, b, c and simplify your answer.

The roots of the quadratic equation, $ax^2+bx+c=0$ are the (possibly complex) numbers $x_1 = \dfrac{-b-\sqrt{b^2-4ac}}{2a}$, $x_2 = \dfrac{-b+\sqrt{b^2-4ac}}{2a}$. In the case at hand, $a=2$, $b=-8$, $c=10$, so the roots are $x_1 = \dfrac{-(-8)-\sqrt{(-8)^2-4\cdot 2\cdot 10}}{2\cdot 2} = \dfrac{8-\sqrt{64-80}}{4}$ $= \dfrac{8-\sqrt{-16}}{4} = \dfrac{8-4i}{4} = 2-i$, and its conjugate $x_2 = \overline{x_1} = 2+i$.

3. (10 points) Find all x that satisfy the equation $\dfrac{x}{x+3} + \dfrac{6}{x^2-9} = \dfrac{2}{x-3} \Leftrightarrow 0 = \dfrac{2}{x-3} - \dfrac{x}{x+3} - \dfrac{6}{x^2-9}$

$= \dfrac{2}{x-3}\cdot\dfrac{x+3}{x+3} - \dfrac{x-3}{x-3}\cdot\dfrac{x}{x+3} - \dfrac{6}{(x-3)(x+3)}$

$= \dfrac{2(x+3)-(x-3)x-6}{(x-3)(x+3)} = \dfrac{2x+6-x^2+3x-6}{(x-3)(x+3)} = \dfrac{-x^2+5x}{(x-3)(x+3)} = \dfrac{-x(x-5)}{(x-3)(x+3)}$

$\Leftrightarrow -x(x-5) = 0 \Leftrightarrow$ either $x=0$ or $x=5$.

[Here "$\Leftrightarrow$" means "logically equivalent to".]

4. (10 points) Find all the real numbers x which satisfy the equation $\sqrt{8+x} + x = 4 \Rightarrow \sqrt{8+x} = 4-x \Rightarrow$

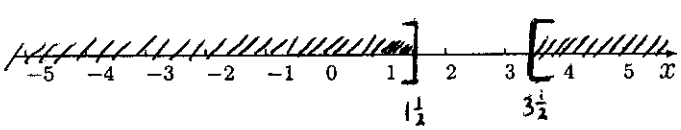
$(\sqrt{8+x})^2 = (4-x)^2 \Rightarrow 8 + x = 16 - 8x + x^2 \Rightarrow 0 = x^2 - 9x + 8 = (x-8)(x-1) \Rightarrow$ either $x=1$ or $x=8$.

[Here "$\Rightarrow$" means "logically implies".] In summary, only two candidates.

Now one checks by back substitution in the original equation that

$\begin{cases} x = 1 & \text{satisfies the original equation.} \\ x = 8 & \text{does not satisfy the original equation.} \end{cases}$

5. (6 points) Solve the inequality $|2x - 5| \geq 2$. Sketch the solution on the real number line below. Remember to indicate the status of endpoints.



$|X| \geq 2$ means X farther from 0 than 2, that is, either $X \leq -2$ or $X \geq 2$

either $2x-5 \leq -2$ or $2x-5 \geq 2$

$2x \leq 3$ $\quad$ $2x \geq 7$

$x \leq 3/2$ $\quad$ $x \geq 7/2$

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Graders: Galstad, Manspeaker, Smith, Steele

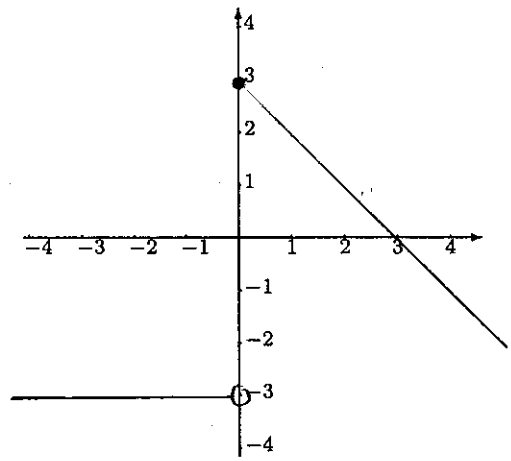
6. (6 points) Solve the inequality $5(x-1) \geq 3 - 7x$. Give your answer in interval notation.

$5x - 5 \geq 3 - 7x$ so $12x \geq 8$ so $x \geq \frac{8}{12} = \frac{2}{3}$

Thus the solution is $[\frac{2}{3}, \infty)$

7. (7 points) Sketch the graph of the piecewise-defined function

$$f(x) = \begin{cases} -x+3, & x \geq 0, \\ -3, & x < 0. \end{cases}$$



8. (8 points) For the graph of the quadratic

$$y = -3x^2 + 12x - 7$$

(a) Find the vertex. Is it a relative maximum or minimum?

$y = -3(x^2 - 4x) - 7 = -3(x^2 - 4x + 4) + 12 - 7 = -3(x-2)^2 + 5$

so $y = a(x-h)^2 + k$ with $a = -3$, $h = 2$ and $k = 5$ and vertex $= (h,k) = (2,5)$.

It is a maximum since $a = -3 < 0$

(b) Find the x-intercepts if any (give exact values not a calculator approximation).

$y = 0 \Leftrightarrow -3(x-2)^2 + 5 = 0 \Leftrightarrow 5 = 3(x-2)^2 \Leftrightarrow \frac{5}{3} = (x-2)^2$

$\Leftrightarrow \pm\sqrt{\frac{5}{3}} = x - 2 \Leftrightarrow x = 2 \pm \sqrt{\frac{5}{3}}$. Thus the x-intercepts are $(2 - \sqrt{\frac{5}{3}}, 0)$ and $(2 + \sqrt{\frac{5}{3}}, 0)$.

9. (6 points) (a) Test to see whether the function $f(x) = \dfrac{x^3}{2x^2+5}$ is even, odd or neither.

For any value of x we have $f(-x) = \dfrac{(-x)^3}{2(-x)^2+5} = \dfrac{-x^3}{2x^2+5} = -f(x)$

so f is $\boxed{\text{odd}}$

(b) Describe the symmetries of the graph of $y^3 = x$.

Since $(-y)^3 = -y^3$ we get that (x,y) satisfies the equation $y^3 = x$ if and only if $(-x,-y)$ satisfies the equation. Thus graph symmetric w.r.t. origin.

Graph not symmetric w.r.t. x-axis and y-axis

10. (6 points) Solve the linear equation for x. Give an exact answer, not a calculator approximation.

$$3(x + \sqrt{5}) - 8 = \sqrt{5}(x+4)$$

$3x + 3\sqrt{5} - 8 = \sqrt{5}x + 4\sqrt{5} \Leftrightarrow (3 - \sqrt{5})x = 8 + \sqrt{5}$

so $\boxed{x = \dfrac{8+\sqrt{5}}{3-\sqrt{5}}}$

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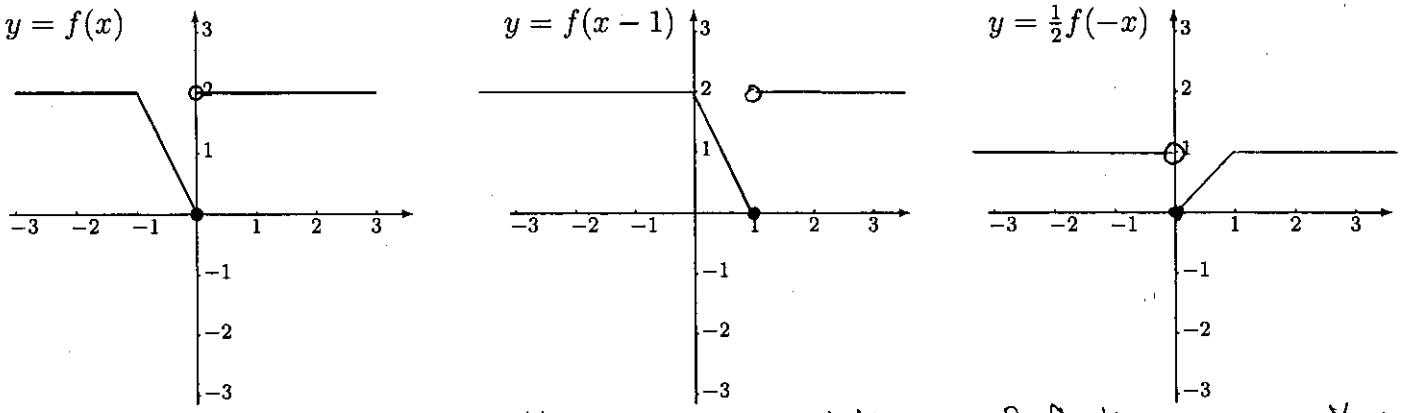
Graders: Hansen, Kearn, Kinkade, Schick

11. (9 points) The graph of $y = f(x)$ is shown on the first set of axes. Sketch the graphs requested on the other two. Underneath describe the transformations in words.



Horizontal Translation Shift 1 unit to the right.

Reflection across y-axis followed by vertical shrinking by factor $\frac{1}{2}$.

12. (8 points)(a) Use your calculator to graph the function

$$f(x) = x^3 + x^2 - 9x + 7$$

from $x = -5$ to $x = 5$ (use zoomfit for the y values and label your axes with the resulting y range).

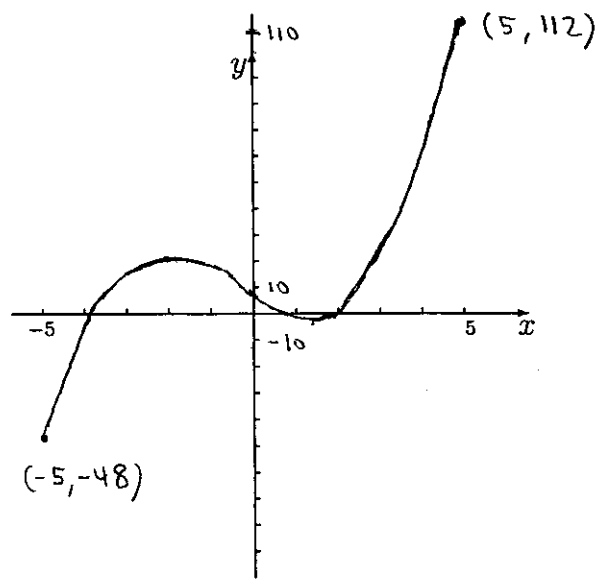
(b) Give the coordinates (to 3 decimal places) of the points:

relative maximum = X = -2.097, Y = 21.049

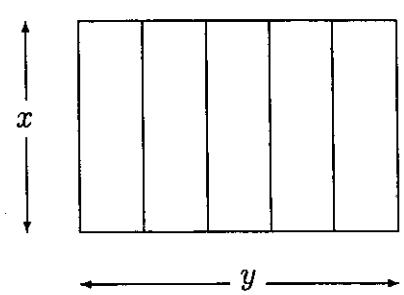
relative minimum = X = 1.431, Y = -0.901

(c) Specify the interval(s) on which the function is increasing.

$(-\infty, -2.097)$ and $(1.431, \infty)$



13. (9 points) A gardener has 2000 yards of fencing to enclose five adjacent plots as shown. Let x denote the width of the enclosure and y its total length.



(a) Write the length y in terms of x:

$2y + 6x = 2000$

so $2y = 2000 - 6x$ so $\boxed{y = 1000 - 3x}$

(b) Write the total area enclosed in terms of x: Area $= xy$ so inserting from (a) gives

$$\text{Area} = x(1000 - 3x) = -3x^2 + 1000x, \quad 0 < x < \frac{1000}{3}$$

(c) What choice of x maximizes this area? Justify your answer. Area $= -3x^2 + 1000x = ax^2 + bx + c$ with $a = -3$, $b = 1000$, $c = 0$. Since $a = -3 < 0$ a maximum is attained at the vertex where $x = -\frac{b}{2a} = -\frac{1000}{2(-3)} = \frac{1000}{6} \approx 166.67$

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