Graders: Burkel, Elston, Hallzuer, Rice

Total/100	Grade

Name:
Recitation Instructor:
Recitation Day & Time:

MATH 100 College Algebra – Exam I

7:15–8:15pm, Tuesday February 7, 2006

Check that that you have all three pages. SHOW ALL YOUR WORK.

1. (6 points) Simplify $\frac{(x^{-1}y^3)^2}{(yx^{-3})^3}$ avoiding negative exponents (you may assume that $x, y \neq 0$):

$\frac{(x^{-1}y^3)^2}{(yx^{-3})^3} = \frac{(x^{-1})^2(y^3)^2}{y^3(x^{-3})^3} = \frac{x^{(-1)2}y^{3\cdot 2}}{y^3x^{(-3)\cdot 3}} = \frac{x^{-2}y^6}{y^3x^{-9}} = x^{-2}x^9y^6y^{-3} = x^{-2+9}y^{6-3} = x^7y^3$

2. (8 points) Write as a rational power of a (assume a is positive):

(i) $\frac{\sqrt[4]{a^2}}{\sqrt[3]{a}} = \frac{(a^2)^{1/4}}{a^{1/3}} = \frac{a^{2\cdot 1/4}}{a^{1/3}} = \frac{a^{1/2}}{a^{1/3}} = a^{1/2-1/3} = a^{1/6}$

(ii) $\sqrt[7]{a^2\sqrt[4]{a}} = (a^2\sqrt[4]{a})^{1/7} = (a^2a^{1/4})^{1/7} = (a^{2+1/4})^{1/7} = (a^{9/4})^{1/7} = a^{9/4\cdot 1/7} = a^{9/28}$

3. (7 points) Simplify by extracting squares (y could be negative in (a)):

(a) $\sqrt{y^8} = \sqrt{y^{4\cdot 2}} = \sqrt{(y^4)^2} = y^4$ (no absolute value needed because y^4 cannot be neg.)

(b) $7\sqrt{28} - 3\sqrt{63} = 7\sqrt{4\cdot 7} - 3\sqrt{9\cdot 7} = 7\sqrt{4}\sqrt{7} - 3\sqrt{9}\sqrt{7} = 7\cdot 2\sqrt{7} - 3\cdot 3\sqrt{7} = 14\sqrt{7} - 9\sqrt{7} = 5\sqrt{7}$.

(No decimal answer can be correct because $\sqrt{7}$ is irrational.)

4. (5 points) Multiply out and write the polynomial in standard form: $(2x^2 - x + 7)(2x^2 + x + 7) =$

$([2x^2+7]-x)([2x^2+7]+x) = [2x^2+7]^2 - x^2 = (2x^2)^2 + 2\cdot 7\cdot 2x^2 + 7^2 - x^2 = 4x^4 + 28x^2 + 49 - x^2$

$(A-x)(A+x) = A^2 - x^2$ difference of squares

$= 4x^4 + 27x^2 + 49$

5. (6 points) Simplify: $\frac{x^2 - 5x - 14}{2x^2 - 14x} = \frac{(x-7)(x+2)}{2x(x-7)} = \frac{x+2}{2x}$. The numerator was factored by the method of "educated guessing", as in Examples 3, 4 p. 20 of text.

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Graders: Pinner, Galstad, Manspeaker, Steele

6. (12 points) Factor the following polynomials as far as possible:

(a) $3x^2 + x - 2 =$ $(3x-2)(x+1)$

(b) $25x^2 - 16 =$ $(5x)^2 - 4^2 = (5x-4)(5x+4)$

(c) $4(x-6)^3 + (x+1)(x-6)^2 =$ $(x-6)^2\left(4(x-6) + (x+1)\right)$
$= (x-6)^2(4x-24+x+1) = (x-6)^2(5x-23)$

7. (7 points) Simplify: $\dfrac{\dfrac{5}{a+2} - \dfrac{4}{a+1}}{\dfrac{a+2}{a+1}}$.

$= \dfrac{\dfrac{5(a+1) - 4(a+2)}{(a+2)(a+1)}}{\dfrac{a+2}{a+1}} = \dfrac{5a+5-4a-8}{(a+2)(a+1)} \dfrac{(a+1)}{(a+2)}$

$= \dfrac{(a-3)\cancel{(a+1)}}{(a+2)\cancel{(a+1)}(a+2)} = \dfrac{(a-3)}{(a+2)^2}$ $a \neq -1$

8. (7 points) Perform the addition and simplify: $\dfrac{4}{z^2-9} + \dfrac{2}{z^2-9z+18}$.

$\dfrac{4}{(z-3)(z+3)} + \dfrac{2}{(z-3)(z-6)} = \dfrac{4(z-6) + 2(z+3)}{(z-3)(z+3)(z-6)} = \dfrac{4z-24+2z+6}{(z-3)(z+3)(z-6)}$

$= \dfrac{6z-18}{(z-3)(z+3)(z-6)} = \dfrac{6\cancel{(z-3)}}{\cancel{(z-3)}(z+3)(z-6)} = \dfrac{6}{(z+3)(z-6)}$

$z \neq 3$

9. (8 points) Use your calculator to graph

$$y = x^3 - x^2 - 6x + 1,$$

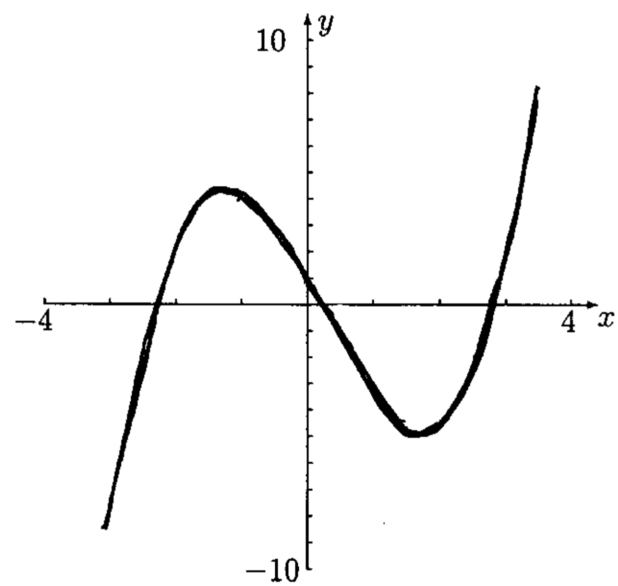
with the window settings shown.
Find the coordinates of any intercepts correct to three decimal places:

x-intercepts= (0.163,0) (2.931,0) (-2.094,0)

y-intercept= (0,1)

x=0

using $\boxed{\text{2nd}}$ $\boxed{\text{CALC}}$ $\boxed{\text{2:ZERO}}$



$\boxed{\text{WINDOW}}$ Xmin = −4
Xmax = 4
Ymin = −10
Ymax = 10

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Graders: Hansen, Kearn, Kinkade, Schick

10. (9 points) The points $(3,-1)$ and $(-1,1)$ are endpoints of the diameter of a circle.

(a) What is the length of the diameter? Let $(x_1,y_1)=(3,-1)$ and $(x_2,y_2)=(-1,1)$ Then

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(-1-3)^2+(1-(-1))^2}=\sqrt{(-4)^2+2^2}=\sqrt{16+4}=\sqrt{20}$$

(b) What is the center of the circle?

$$(h,k)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{3+(-1)}{2},\frac{(-1)+1}{2}\right)=\left(\frac{2}{2},\frac{0}{2}\right)=(1,0)$$

(c) Write down an equation of the circle.

$(x-h)^2+(y-k)^2=r^2$ where $r=\frac{d}{2}$ so $r^2=\left(\frac{d}{2}\right)^2=\frac{d^2}{2^2}=\frac{20}{4}=5$

so equation is $(x-1)^2+(y-0)^2=5$

so $(x-1)^2+y^2=5$

11. (5 points) Rationalize the numerator: $\dfrac{8+\sqrt{3}}{6-\sqrt{3}}$.

$$\frac{8+\sqrt{3}}{6-\sqrt{3}}=\frac{(8+\sqrt{3})(8-\sqrt{3})}{(6-\sqrt{3})(8-\sqrt{3})}=\frac{8^2-(\sqrt{3})^2}{6\cdot 8-6\sqrt{3}-8\sqrt{3}+(\sqrt{3})^2}=\frac{64-3}{48-14\sqrt{3}+3}=\frac{61}{51-14\sqrt{3}}$$

12. (6 points) (a) The slope of the line passing through $(-2,8)$ and $(4,2)$ is $m=$ $\underline{-1}$.

Let $(x_1,y_1)=(-2,8)$ and $(x_2,y_2)=(4,2)$. Then

$$m=\frac{\Delta y}{\Delta x}=\frac{y_2-y_1}{x_2-x_1}=\frac{2-8}{4-(-2)}=\frac{-6}{6}=-1$$

(b) Find an equation for the line passing through the two points in part (a).

$y=mx+b=(-1)x+b=-x+b$. We know that $(x,y)=(-2,8)$ is a point on the line so $8=-(-2)+b=2+b$ so $b=6$. Thus $y=-x+6$.

13. (6 points) (a) The slope of the line $2x+5y=1$ is $m_1=$ $\underline{-2/5}$.

$$2x+5y=1 \iff 5y=-2x+1 \iff y=\frac{-2x}{5}+\frac{1}{5}=\underbrace{-\frac{2}{5}}_{\text{slope}}x+1$$

(b) The slope of a line perpendicular to the line in (a) is $m_2=$ $\underline{5/2}$.

We have $m_1m_2=-1$ so $\left(-\frac{2}{5}\right)m_2=-1$ so $m_2=\frac{5}{2}$

14. (4 points) What is the domain of the function $f(x)=\dfrac{x^2+x}{x^2+5x-6}$?

Since $x^2+5x-6=(x+6)(x-1)=0$ if and only if $x=-6$ or $x=1$

the domain $=\{x\in\mathbb{R} \mid x\neq -6 \text{ and } x\neq 1\}$

15. (4 points) Let $g(y)=y^2+3y-2$. Find and simplify $g(y-4)$.

$$g(y-4)=(y-4)^2+3(y-4)-2=y^2-8y+16+3y-12-2=y^2-5y+2$$

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