Graders: Burckel, Elston, Rice

Name:

Recitation Instructor:

Recitation Day & Time:

Total/200	Grade

MATH 100 College Algebra – Final Exam

7:00–8:50pm, Wednesday May 10, 2006

Check that you have all five pages. SHOW ALL YOUR WORK. Write complex answers in $a + bi$ form.

1. (6 points) Perform the division: $(3x^3 + 5x^2 - 7) \div (x - 2)$:

Quotient= $3x^2+11x+22$, Remainder= 37.

Synthetic division:

$$\begin{array}{r|rrrr} 2 & 3 & 5 & 0 & -7 \\ & & 6 & 22 & 44 \\ \hline & 3 & 11 & 22 & 37 \end{array}$$

or

$$\begin{array}{r} 3x^2+11x+22 \\ x-2\,\overline{)\,3x^3+5x^2+0x-7} \\ 3x^2-6x^2 \\ \hline 11x^2+0x \\ 11x^2-22x \\ \hline 22x-7 \\ 22x-44 \\ \hline 37 \end{array}$$

2. (12 points) $f(x) = 2x^3 - x^2 + 18x - 9$.

(a) Use the rational-zeros test to find a rational zero of the polynomial f. List all the *possible* rational zeros given by the test, as well as those which actually are zeros.

Possible rational zeros $\frac{p}{q} = \frac{\pm 1, \pm 3, \pm 9}{\pm 1, \pm 2} = \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 3, \pm\frac{9}{2}, \pm 9$

(b) Find the remaining zeros of f (including complex zeros). $x = \frac{1}{2}$ only actual rational zero of f.

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -1 & 18 & -9 \\ & & 1 & 0 & 9 \\ \hline & 2 & 0 & 18 & 0 \end{array}$$

So $f(x) = (x - \frac{1}{2})(2x^2+18)$

$= (2x-1)(x^2+9) = (2x-1)(x-3i)(x+3i)$

so remaining zeros $= \pm 3i$

3. (10 points) Factor into linear factors (using complex numbers where necessary):

(a) $4x^2 + 10x - 6 = (2x+6)(2x-1)$

(b) $x^4 + x^2 - 12 = (x^2+4)(x^2-3) = (x+2i)(x-2i)(x+\sqrt{3})(x-\sqrt{3})$

4. (10 points) (a) Find the slope of the line passing through $(2, 3)$ and $(-1, 6)$.

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{6-3}{-1-2} = \frac{3}{-3} = -1$$

(b) An equation for the line in (a) is $y =$ $-x+5$.

From a) $y = -x + b$. Since $(2,3)$ is a point on the line

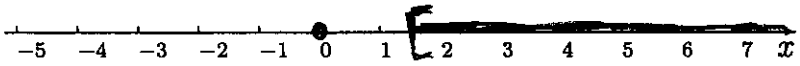
$3 = -2 + b \Rightarrow b = 5$

PG SCORE/38

Graders : Pinner, Galstad, Manspeaker

5. (8 points) Solve the polynomial inequality $3x^2 \leq 2x^3$. Sketch the solution below (remember to indicate the status of endpoints).

$x=0$ is a solution.



For $x \neq 0$, $3x^2 \leq 2x^3 \Leftrightarrow \frac{3}{2} \leq x$. Thus the solution is

$\{0\} \cup [\frac{3}{2}, \infty)$

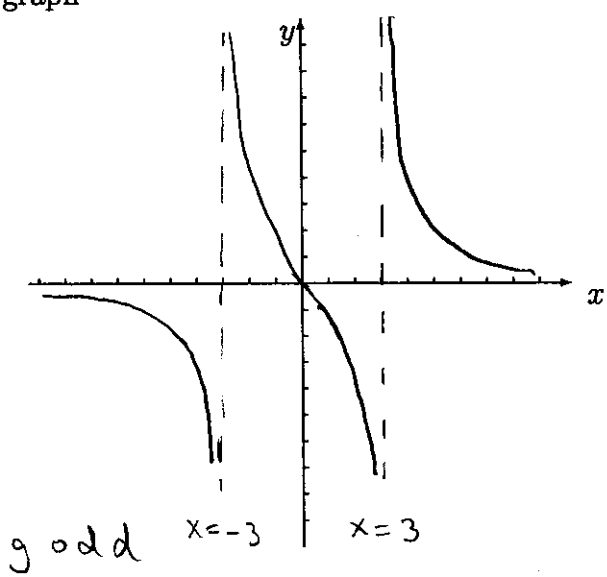
6. (10 points) Find all x satisfying the equation: $\dfrac{4}{x^2-3x} - \dfrac{4}{x} = \dfrac{2x}{x-3}$. $\Leftrightarrow \dfrac{4}{x^2-3x} - \dfrac{4(x-3)}{x^2-3x} = \dfrac{2x^2}{x^2-3x}$

$\Leftrightarrow 0 = \dfrac{2x^2-4+4(x-3)}{x^2-3x} \Leftrightarrow 2x^2+4x-16=0 \Leftrightarrow x^2+2x-8=0$

$\Leftrightarrow (x+4)(x-2)=0 \Leftrightarrow \boxed{x=-4,\ x=2}$ (We have to assume all over that $x \neq 0$ and $x \neq 3$)

7. (12 points) Provide the following information and use it to graph

$$g(x) = \frac{3x}{x^2-9}.$$



(a) The x-intercept(s) and y-intercept(s):

(0,0) is only x-intercept and also only y-intercept

(b) Equations for all the vertical asymptotes:

$x^2-9=(x-3)(x+3)$; so $x=3$ and $x=-3$ are the vertical asymptotes

(c) Equation of the horizontal asymptote:

$y=0$ horizontal asymptote

(d) Symmetry: Is $g(x)$ even, odd or neither?

$g(-x) = \dfrac{3(-x)}{(-x)^2-9} = -\dfrac{3x}{x^2-9} = -g(x)$ so g odd

8. (12 points) For the following functions find the vertical, horizontal and oblique asymptotes (write an **equation** for the asymptote in the space provided or N/A if there is not an asymptote of that type):

	vertical	horizontal	oblique
$f(x) = \dfrac{x^2}{x-8}$	$x=8$	N/A	$y=x+8$
$g(x) = \dfrac{3x+2}{2x+4}$	$x=-2$	$y=\frac{3}{2}$	N/A

$$\begin{array}{r|l} & x+8 \\ \hline x-8 & x^2+0x+0 \\ & x^2-8x \\ \hline & 8x+0 \\ & 8x-64 \\ \hline & 64 \end{array}$$

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Graders: Smith, Steele

9. (9 points) Solve the inequality $|2x-1| \geq 1$. Give your answer in interval notation.

$|2x-1| \geq 1$

$\Updownarrow$

$2x-1 \leq -1$ or $2x-1 \geq 1$

$\Updownarrow$

$2x \leq 0$ or $2x \geq 2$

$\Updownarrow$

$x \leq 0$ or $x \geq 1$

so solution is

$(-\infty, 0] \cup [1, \infty)$

10. (10 points) Find all the real numbers x which satisfy the equation $\sqrt{x+6} + x = 0$.

$\sqrt{x+6} + x = 0 \iff \sqrt{x+6} = -x \rightarrow x+6 = (-x)^2$

$\iff 0 = x^2 - x - 6 \iff 0 = (x-3)(x+2) \iff x = -2$ or $x = 3$

Check: $x = -2$: $\sqrt{x+6} + x = \sqrt{-2+6} + (-2) = \sqrt{4} - 2 = 0$ ✓

$x = 3$: $\sqrt{x+6} + x = \sqrt{9} + 3 = 6$ ✗

so $x = -2$ only solution

11. (12 points) (a) Simplify: $\dfrac{\dfrac{5}{x+2} + \dfrac{3}{x}}{\dfrac{4x+3}{x+2}}$.

$$\frac{\dfrac{5x+3(x+2)}{x(x+2)}}{\dfrac{4x+3}{x+2}} = \frac{8x+6}{x(x+2)} \cdot \frac{x+2}{4x+3} = \frac{2(4x+3)}{x(x+2)} \cdot \frac{(x+2)}{(4x+3)} = \frac{2}{x}$$

(b) Perform the subtraction and simplify: $\dfrac{y}{y^2-y-20} - \dfrac{2}{y+4}$.

$$\frac{y}{y^2-y-20} - \frac{2}{y+4} = \frac{y}{(y-5)(y+4)} - \frac{2(y-5)}{(y-5)(y+4)} = \frac{y-2(y-5)}{y^2-y-20} = \frac{10-y}{y^2-y-20}$$

12. (10 points) Simplify so that only positive rational exponents occur (assume that x and y are positive):

(a) $\dfrac{(x^2y^2)^3}{xy^3} = \dfrac{x^6y^6}{xy^3} = x^5y^3$

(b) $\dfrac{\sqrt[3]{y^4}}{\sqrt{y}} = \dfrac{y^{4/3}}{y^{1/2}} = \dfrac{y^{8/6}}{y^{3/6}} = y^{\frac{8}{6}-\frac{3}{6}} = y^{5/6}$

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Graders: Kearn, Kinkade, Manspeaker

13. (8 points) (a) Find the critical values for the inequality $2 \le \frac{5}{x-1}$. $\Leftrightarrow$ $0 \le f(x)$

Where $f(x) = \frac{5}{x-1} - 2 = \frac{5-2(x-1)}{x-1} = \frac{7-2x}{x-1}$ is the related function.

The critical values are $\boxed{x=1}$ (where $f(x)$ is not defined) and $\boxed{x=\frac{7}{2}}$ (where $f(x)=0$).

(b) Solve the inequality in (a). Sketch the solution below (remember to indicate the status of endpoints).

Interval, Test value

$(-\infty, 1)$, $f(0) = -7 < 0$

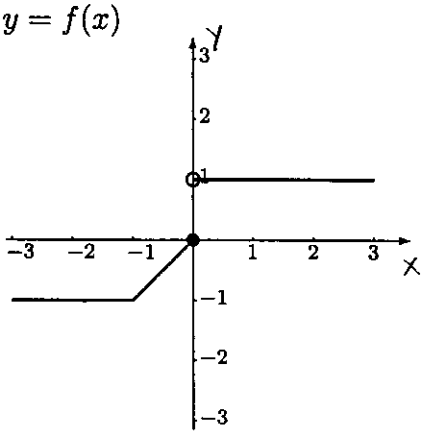
$(1, \frac{7}{2})$, $f(2) = 3 > 0$

$(\frac{7}{2}, \infty)$, $f(4) = -\frac{1}{3} < 0$

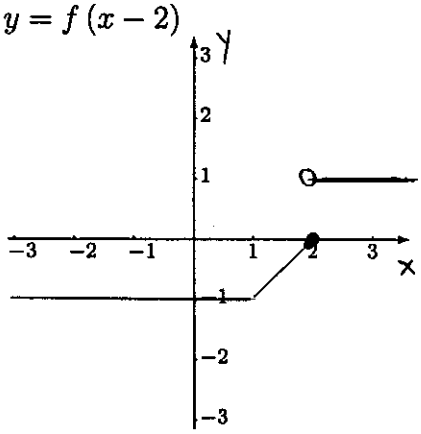
Thus solution is $(1, \frac{7}{2}]$.

14. (9 points) The graph of $y = f(x)$ is shown in the first picture. Sketch the graphs requested on the other two. Underneath describe in words the transformations of the graph of $y = f(x)$.

$y = f(x)$

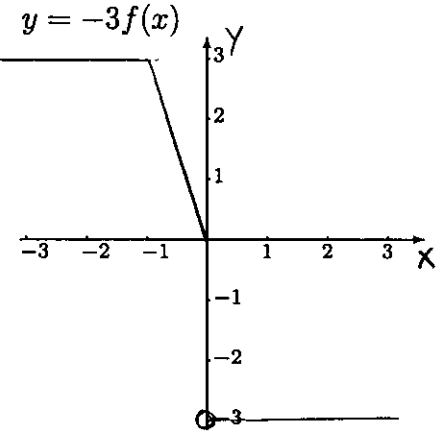


$y = f(x-2)$



Horizontal translation 2 units to the right

$y = -3f(x)$



reflection across x-axis and vertical stretching by factor 3

15. (16 points) (a) Write as a single logarithm: $\frac{1}{4}\ln x + \ln y - 5\ln z =$ $\ln\left(\frac{x^{1/4}y}{z^5}\right)$

(b) Express $\log_2 18$ in terms of natural logarithms. Use the calculator to evaluate it to 3 decimal places.

$\log_2 18 = \frac{\ln(18)}{\ln(2)} = 4.1699\ldots \approx 4.170$

(c) Solve the logarithmic equation $\log_4(x-3) - \log_4(x) = 1$.

$\log_4$ is increasing so $\log_4(x-3) - \log_4(x)$ is negative for all allowed inputs x (i.e. for all $x > 3$) so

No solutions

16. (8 points) Use either the method of elimination or that of substitution to solve the system of equations

$$3x + 4y = 10$$
$$x - y = 1.$$

By 2nd eq. $x = y+1$. Substituting into 1st eq. gives

$3(y+1) + 4y = 10 \Leftrightarrow 7y + 3 = 10 \Leftrightarrow y = 1$

But then $x = y+1 = 1+1 = 2$ so $\boxed{(x,y) = (2,1)}$

PG SCORE/41

Graders: Schick, Hallauer, Rice

17. (12 points) (a) If $f(x) = x^2 + 5$ and $g(x) = x - 2$, then the composition $(f \circ g)(x) =$

$(f \circ g)(x) = f(g(x)) = g(x)^2 + 5 = (x-2)^2 + 5 = x^2 - 4x + 4 + 5 = x^2 - 4x + 9$

(b) If $h(x) = 2x^3 + 6$, then the inverse function $h^{-1}(x) = \sqrt[3]{\frac{x-6}{2}}$

1) $y = 2x^3 + 6$ 2) Interchange x and y to get $x = 2y^3 + 6$

3) Solve for y to get $\frac{x-6}{2} = y^3 \Leftrightarrow y = \sqrt[3]{\frac{x-6}{2}}$ 4) Write $h^{-1}(x)$ for y.

18. (10 points) A savings account offers an annual rate of 5% compounded continuously.

(a) If \$8,000 is deposited in the account, what will be the balance after 5 years?

$P(t) = 8{,}000\, e^{0.05t}$, $P(5) = 8{,}000 \cdot e^{0.05 \cdot 5} = 8{,}000 \cdot e^{0.25}$

$= 10{,}272.203\ldots \approx 10{,}272.20$ (\$)

(b) What is the tripling time (i.e., the time for the amount in the account to triple)?

$P(t) = 3 \cdot 8000 \Leftrightarrow e^{0.05t} = 3 \Leftrightarrow 0.05t = \ln(3)$

$\Leftrightarrow t = \frac{\ln(3)}{0.05} = 21.972\ldots \approx 22$ years

19. (8 points) (a) Give the augmented matrix for the system of equations:

$$\begin{aligned} 3x - 2y + 4z &= 15 \\ 2x + 2y + z &= 30 \\ x - 3y + z &= 3. \end{aligned} \qquad \left[\begin{array}{ccc|c} 3 & -2 & 4 & 15 \\ 2 & 2 & 1 & 30 \\ 1 & -3 & 1 & 3 \end{array}\right]$$

(b) Use your calculator to find the reduced row-echelon form of the matrix in (a).

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 17 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -8 \end{array}\right]$$

(c) Use (b) to solve the system of equations in (a): $x =$ 17, $y =$ 2, $z =$ -8.

20. (8 points) (a) Write the system below as a matrix equation $AX = B$

$$\begin{aligned} 2x + 3y + 2z &= -2 \\ 3x + 3y + 4z &= 1 \\ -x - y - z &= 2. \end{aligned} \qquad \begin{bmatrix} 2 & 3 & 2 \\ 3 & 3 & 4 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

(b) Use your calculator to find A^{-1}, the inverse of the coefficient matrix A, then $A^{-1}B$.

$$A^{-1} = \begin{bmatrix} -1 & -1 & -6 \\ 1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix} \qquad A^{-1}B = \begin{bmatrix} -11 \\ 2 \\ 7 \end{bmatrix}$$

(c) Use (b) to solve the system of equations in (a): $x =$ -11, $y =$ 2, $z =$ 7.

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