Name:

MATH 512 Intro to Modern Algebra – **Exam II**

Wednesday, October 27, 2004

Check that that you have all four pages - note that the pages are double-sided

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1. (20 points) For the permutations

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 6 & 5 & 1 & 4 & 2 \end{pmatrix}, \qquad \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 1 & 4 & 6 & 2 & 3 \end{pmatrix}.$$

(a) Find $\sigma^{-1}\tau$.

(b) Write σ as a product of disjoint cycles.

(c) Write σ as a product of transpositions.

(d) Is σ even, odd, neither or both?

(e) How many cosets does $H =< (1, 2, 3, 4) >$ have in S_6?

2. (6 points) Find all the abelian groups of order 200, up to isomorphism.

PG SCORE/26

3. (24 points) Circle True (T) or False (F).

T F (a) If $|G| = p$ is prime then G is cyclic.

T F (b) $|\mathbb{Z}_n \times \mathbb{Z}_m| = nm$ iff $gcd(n, m) = 1$.

T F (c) $\mathbb{Z}_9 \times \mathbb{Z}_6$ has a subgroup of order 18.

T F (d) $\mathbb{Z}_{15} \times \mathbb{Z}_{14} \simeq \mathbb{Z}_{10} \times \mathbb{Z}_{21}$.

T F (e) The element $(2, 5)$ has infinite order in $\mathbb{Z} \times \mathbb{Z}_{15}$.

T F (f) If G is a finite group and $K < H < G$ then $(G : K) = (G : H)(H : K)$.

T F (g) In a finite group the order of a^k divides the order of a.

T F (h) The odd permutations in S_n form a subgroup of order $\frac{1}{2}n!$

T F (i) If G is a group of order 100 then G has no subgroup of order 3.

T F (j) If a has order 50 then $< a^{15} >=< a^5 >$.

T F (k) If σ is a cycle then σ^2 is a cycle.

T F (l) The dihedral group $D_4 \simeq \mathbb{Z}_2 \times \mathbb{Z}_4$.

4. (10 points) (a) Define what it means for a subgroup H of a group G to be a normal subgroup of G.

$*$	e	a	b	c	d	f
e	e	a	b	c	d	f
a	a	b	e	f	c	d
b	b	e	a	d	f	c
c	c	d	f	e	a	b
d	d	f	c	b	e	a
f	f	c	d	a	b	e

For the group $G = \{e, a, b, c, d, f\}$ given by the table:

(b) Find the left cosets of $H = \{e, c\}$ in G.

(c) Find the right cosets of $H = \{e, c\}$ in G.

(d) Is $H = \{e, c\}$ a normal subgroup of G?

PG SCORE/34

5. (20 points) (a) What is the order of 35 in $\mathbb{Z}_{100}$?

(b) What is the subgroup of $\mathbb{Z}$ generated by the set $\{35, 21\}$?

(c) What is the subgroup of $\mathbb{Z}_{100}$ generated by the set $\{35, 21\}$?

(d) What is the order of the element $(2, 6)$ in the group $\mathbb{Z}_{30} \times \mathbb{Z}_{40}$?

(e) Prove that $\mathbb{Z}_6 \times \mathbb{Z}_9$ is not cyclic.

6. (8 points) (a) State Lagrange's Theorem.

(b) Suppose that G is a non-cyclic group of order 8. Show that $a^4 = e$ for all a in G.

PG SCORE/28

7. (6 points) Prove that a cyclic group is abelian.

8. (6 points) Suppose that H is a subgroup of G, and that $a, b \in G$. Prove that if $a^{-1}b \in H$ then $bH = aH$.

PG SCORE/12