Name:

Total/100

MATH 510 Discrete Math – **Exam I**

Wednesday, June 30, 2004

Check that that you have all three pages - page two is on the back of page one.
Show all your work and reasoning. Evaluate factorials and binomial coefficients only if you have sufficient time.
If you apply the box principle make sure you indicate clearly what boxes you are using.

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1. (20 points) A 6 digit password is to be made using only the eight digits $1, 2, 3, 4, 5, 6, 7$ or 8. How many passwords are possible under the following assumptions:

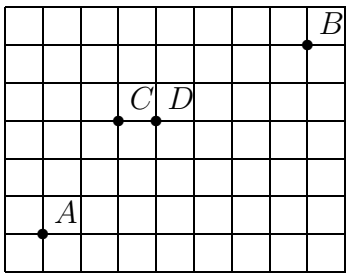
(a) Repeat digits are allowed.

(b) No digit is repeated.

(c) The number does not begin 785.

(d) The password contains three fives (and three non-fives).

2. (5 points) The streets of a city run North-South and East-West. Assuming that you always take the shortest route, how many routes are there from A to B which pass along the street CD?



3. (5 points) Count the number of integer solutions to $x_1 + x_2 + x_3 = 10$ with $x_1 \geq 3$, $x_2 \geq 0$, $x_3 \geq 0$.

PG SCORE/30

4. (5 points) Eight colored balls, 3 blue, 4 green and 1 red, are arranged in a line. How many arrangements are possible (assume that balls of the same color are identical)?

5. (5 points) A company has 10 job applicants from which to choose 3 programming trainees and 4 sales trainees. If only 6 of the applicants are qualified for the programming positions, how many ways can they fill the 7 traineeships? Assume that no one is given more than one traineeship.

6. (5 points) How many ways can 6 non-attacking rooks be placed on a 8×8 chess-board?

7. (10 points) Six people are seated around a circular table.

(a) How many circular arrangements are possible?

(b) What if Wanda refuses to sit next to or opposite her ex-husband Vincent?

8. (10 points) A florist has 9 daffodils, 9 gladioli and 9 tulips (assume that flowers of the same type are indistinguishable). How many different bouquets can he make under the following assumptions:

(a) The bouquet contains exactly 9 flowers.

(b) The bouquet can contain any number of flowers (except zero flowers!).

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9. (5 points) Give a **combinatorial** explanation of $\binom{100}{10} = \binom{99}{10} + \binom{99}{9}$.

10. (5 points) Bob chooses 11 (distinct) numbers from 1 to 20. Prove that he has chosen two numbers whose difference is 10.

11. (10 points) There are 6 people at a dinner party.

(a) Prove that two of them must have the same number of acquaintances in the group.

(b) What does Ramsey's Theorem tell you?

12. (5 points) What is the coefficient of x^4y^3 in the expansion of $(2x - 3y)^6$?

13. (10 points) Use the binomial theorem to evaluate the sums:

(a) $\displaystyle\sum_{j=0}^{n} \binom{n}{j} (-1)^{n-j} 5^j$

(b) $\displaystyle\sum_{j=0}^{n} \binom{n}{j} \frac{5^{j+1}}{j+1}$

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