Math 510 Exam I

Time: 12:30pm–1:20pm, Wed, February 26, 2003 **Name:**_____

Please read the problems carefully and do **all** you are asked to do. You need to show your work to indicate how you do each problem. Your proofs and explanations should be literally clear.

Total/100	#1/10	#2/20	#3/30	#4/10	#5/10	#6/10	#7/20

1. (10pts) Prove that, among any ten points in a 1×1 square, there are two points whose distance apart is at most $\frac{1}{3}\sqrt{2}$

2. (20pts) Compute the following numbers:
(a). The number of the positive divisors of $15^4 \times 7^7 \times 11^{11}$

(b). The number of ways to place 8 indistinguishable non-attacking rooks on a 12×12 chess-board.

3. (20pts) How many 5-digit odd numbers (00001 is not a 5-digit number) can be written using digits 0, 1, 4, 5, 6, 7, 8, 9 if
(a). each digit can be used as many times as possible?

(b). each digit can be used at most once?

4. (10pts) In how many ways can 15 indistinguishable apples, 2 pears, and 1 orange be distributed to 5 children such that each child has at least one piece of fruit and no child gets both pear and orange or more than one pear?

5. (10pts) Let n be a positive integer. For any sequence of integers $a_1, a_2, \ldots, a_n$ show that either n divides one of them or n divides $a_i - a_j$ for some $i \neq j$.

6. (10pts) In how many ways can six women, five men and a dog sit around a table such that no two women sit next to each other?

7. (10pts) Use the binomial expansion of $(1+x)^n$ to compute the following sums.

(a). $\sum_{k=1}^{n}(-1)^k k \begin{pmatrix} n \\ k \end{pmatrix} 2^{k-1} =$

(b). $\sum_{k=0}^{n} \frac{1}{k+1} \begin{pmatrix} n \\ k \end{pmatrix} 2^{k+1} =$