Name:

Total/100

MATH 510 Discrete Math – **Final Exam**

Friday, July 30, 2004

Check that that you have all four pages. Show all your work and reasoning.

1. (8 points) The recurrence

$$h_n = 4h_{n-1} + 5h_{n-2}, \;\; h_0 = 5, \; h_1 = 13$$

has generating function

$$g(x) = \frac{5 - 7x}{1 - 4x - 5x^2}.$$

Use the generating function to solve for h_n.

2. (12 points) Determine all the connected, non-isomorphic, **general** graphs with degree sequence 4,4,4.

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3. (8 points) Twelve Twix and two Snickers bars are to be distributed amongst five children. How many ways can this we done so that everyone gets at least one candy bar and no one gets both Snickers?

4. (8 points) Five houses (in a row) are to be painted red, green, yellow or blue. How many ways can this be done under the following assumptions:

(a) No two neighboring houses are painted the same color.

(b) There are two red houses, two yellow houses and one blue house.

5. (6 points) Six people are to be seated around a circular table. How many circular arrangements are possible if Anne and Carol refuse to sit next to Bob?

6. (7 points) Use the binomial theorem $(1+x)^n = \sum_{j=0}^{n}$ __________ to evaluate the sum $\sum_{j=0}^{n} j\binom{n}{j}2^{j-1}$

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7. (8 points) Use the deferred acceptance algorithm to find the men-optimal stable complete marriage for the preferential ranking matrix

$$\begin{bmatrix} 3,3 & 2,1 & 1,4 & 4,2 \\ 3,1 & 1,2 & 4,2 & 2,3 \\ 2,2 & 3,3 & 4,3 & 1,4 \\ 4,4 & 2,4 & 1,1 & 3,1 \end{bmatrix}$$

Here rows A, B, C, D are the women and columns a, b, c, d are the men.

8. (8 points) Use the inclusion-exclusion principle to count the number of integers between 1 and 10,000 that are not divisible by 10,12 or 15.

9. (5 points) Bob picks 7 integers from 1 to 12. Prove that he must have picked a pair of numbers a, b with a dividing b.

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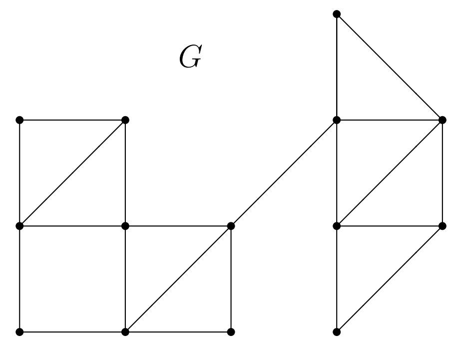
10. (16 points) (i) Does G have a closed Euler trail? Explain



(ii) Does G have an open Euler trail? If so circle the start and finish of the trail.

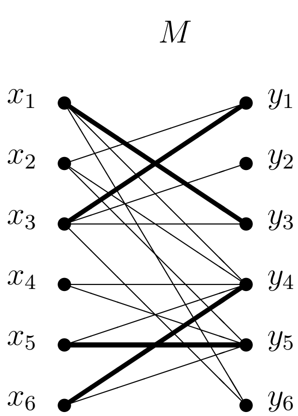
(iii) Show that G has a Hamilton Chain.

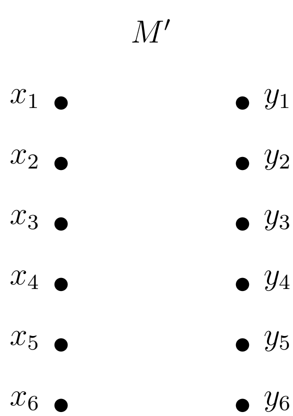
(iv) Draw a spanning subgraph G' of G which is also a tree.

11. (5 points) Evaluate the coefficient of x^3 in the Newton binomial expansion of $(1+x)^{-\frac{1}{2}}$?

12. (9 points) For the bipartite graph and matching M:

(a) Find an M-alternating chain and hence a new matching M' with 5 edges.





(b) Show that its a max matching by finding a cover S with 5 vertices:

$S = \{ \qquad\qquad\qquad\qquad \}$

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