Name:

MATH 506 Number Theory – **Exam II**

Wednesday March 26, 2008

Check that that you have all three pages - note that page two is on the back of page one

---

1. (22 points) Find the following where $a = 2^2 3^2 5^2 13^7$ and $b = 2^5 3^3 13^5 19$:

(a) The prime factorizations of $gcd(a, b)$ and $lcm(a, b)$.

(b) If $2^e || 7a^3 + 20b$ then $e =$ ________.

(c) If $5^f || 1000!$ then $f =$ ________.

(d) If $2^g || 1234567891011151617181920212 2124$ then $g =$ ________.

(e) If $12700140x3243243$ leaves remainder 3 when divided by 11 then the missing digit $x =$ ________.

2. (12 points) (a) Evaluate $\tau(3500)$.

(b) Evaluate $\sigma(3500)$.

(c) What prime factorizations are possible for n if $\tau(n) = 15$?

3. (7 points) Give a proof that there are infinitely many primes.

PG SCORE/41

4. (12 points) (a) Find **all** the right-angled triangles with **coprime** integer sides and base of length 40:

(b) Find **all** the right-angled triangles with **coprime** integer sides and base of length 35:

5. (7 points) Prove that the Fibonacci numbers satisfy $f_n < \left(\frac{\sqrt{5}+1}{2}\right)^n$ for all positive integers n.

6. (6 points) Sieve out the primes from 236 to 253. Primes =

236 237 238 239 240 241 242 243 244 245 246 247 248 249 250 251 252 253

Multiples of which numbers had to be sifted out?

PG SCORE/25

7. (7 points) Prove that $2^{n+2}|(2n+3)!$ for all positive integers n.

8. (20 points) Circle True (T) or False (F).

T F (a) If n is not divisible by any primes $p \leq \sqrt[4]{n}$ then n contains at most three primes.

T F (b) The prime number theorem states that $\lim_{x\to\infty} \frac{\pi(x)}{\ln x} = 1$.

T F (c) There is no pythagorean triple $x^2 + y^2 = z^2$ with $x = 10$.

T F (d) The divisor function satisfies $\tau(ab) \leq \tau(a)\tau(b)$.

T F (e) If p is a prime and $p^2|a^3$ then $p^2|a$.

T F (f) If $3 \nmid a$ then $\tau(3a) = 2\tau(a)$.

T F (g) If $n \leq 288$ is not divisible by any $d \leq 15$ then n is prime.

T F (h) If $f(n)$ is multiplicative with $f(3) = 5$, $f(6) = 10$, $f(10) = 6$ then $f(15) = 15$.

T F (i) If $p^i||a$ and $p^i||(a+b)$ then $p^{2i}||ab$.

T F (j) If k is a multiple of 3 then $3||\underbrace{555...5}_{k \text{ times}}$.

9. (7 points) Prove that $\sqrt[3]{\frac{4}{9}}$ is irrational. (Give a proof, don't just state a theorem).

PG SCORE/34