Name:

MATH 506 Number Theory – **Exam I**

Wednesday February 13, 2008

Check that that you have all three pages - note that page two is on the back of page one

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1. (6 points) Find integers q and r such that $-54 = 13q + r$ with $0 \le r < 13$.

2. (12 points) (a) Use the Euclidean algorithm to compute the greatest common divisor of 3618 and 1139.

(b) Write this gcd as a linear combination of 3618 and 1139.

3. (8 points) Prove that $\dfrac{n^3 + 2n}{3}$ is an integer for all integers n.

4. (6 points) What is the least residue of 776^{79} modulo 7?

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5. (8 points) Use the definition of congruence to prove that if $a \equiv 3 \pmod 4$ and $b \equiv 1 \pmod 4$ then $a^2 + 2ab \equiv 7 \pmod 8$.

6. (12 points) Decide whether the following linear Diophantine equations have any solutions. If so give the general solution, if not say why there are no solutions. There is no need to go through the Euclidean algorithm to find obvious gcds or solutions.

(a) $21x - 35y = 77, \quad x, y \in \mathbb{Z}$.

(b) $24x + 39y = 26, \quad x, y \in \mathbb{Z}$.

7. (10 points) What values can $(3a + 5, 5a + 7)$ take? When is it 4?

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8. (8 points) Use the definition of divisibility to prove that if a, b, c and d are non-zero integers with $a \mid b$ and $b \mid c$ and $c \mid d$ then $a \mid 2c + d$.

9. (22 points) Circle True (T) or False (F).

T F (a) If $(3a, 3b) = 3$ then a and b must be relatively prime.

T F (b) If $n^2 + 1$ is not prime then n is odd.

T F (c) If $a \mid b$ then $a^3 \mid b^3$.

T F (d) If $a \mid b^2$ then $a \mid b$.

T F (e) If $d \mid a$ and $d \mid b$ then $d \mid (a, b)$.

T F (f) Any even integer can be written as a linear combination of 10 and 21.

T F (g) The smallest positive integer of the form $6x + 9y + 15z$, $x, y, z \in \mathbb{Z}$, is 3.

T F (h) For any integer b there exist unique integers q, r with $b = 7q + r$ and $0 < r < 7$.

T F (i) If a and b are both even then $[a, b] = ab/2$.

T F (j) $\{5, 11, -3, -1, 2\}$ is a complete system of residues modulo 5.

T F (k) $2^{66} + 1$ is divisible by $2^{11} + 1$.

10. (8 points) Suppose that a, b and c are positive integers with $b \mid c$ and $m = [a, c]$. Prove that $[a, b] \leq m$.

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