Name:

MATH 506 Number Theory – **Exam II**

Wednesday March 29, 2006

Check that that you have all three pages - note that page two is on the back of page one

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1. (12 points) Find the following:

(a) The prime factorization of $gcd(a, b)$ if $a = 2^5 3^2 11^2 13^7$ and $b = 2^2 3^3 13^5 19$.

(b) The value of e if $3^e || 100!$

(c) The value of f if $2^f || 12345678910111213141516$

2. (14 points) (a) Evaluate $\tau(2200)$.

(b) Evaluate $\sigma(2200)$.

(c) What prime factorizations are possible for n if $\tau(n) = 21$?

(d) What is the smallest n with $\tau(n) = 21$?

3. (8 points) Prove that there are infinitely many primes of the form $4k + 3$.

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4. (12 points) Find all the primitive Pythagorean triples containing:

(a) 20

(b) 65

5. (10 points) a) Suppose that n is a composite number not divisible by any prime $p \leq \sqrt[3]{n}$. Show that n must be a product of two (not necessarily distinct) primes.

b) Give (with justification) a non-trivial factor of $2^{48} + 1$.

6. (6 points) Sieve out the primes from 213 to 230. Primes =

213 214 215 216 217 218 219 220 221 222 223 224 225 226 227 228 229 230

Multiples of which numbers had to be sifted out?

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7. (10 points) The integer $n = 1270014010x603603009009$ has a missing digit x.

(a) What is x if n is a multiple of 7? (Be sure to show the divisiblity test involved).

(b) What is x if n leaves remainder 2 when divided by 9? (Show the test involved).

8. (20 points) Circle True (T) or False (F).

T F (a) If $\sigma(n) > n+1$ then n is composite.

T F (b) If $3^2||a$ and $3^2||b$ then $3^2||(a+b)$.

T F (c) The prime number theorem states that $\lim_{x\to\infty} \frac{\pi(x)}{\frac{\ln x}{x}} = 1$.

T F (d) $2^n - 1$ is prime if n is prime.

T F (e) The divisor function satisfies $\tau(ab) = \tau(a)\tau(b)$ for any positive integers a, b.

T F (f) If $\gcd(a,b) = 1$ and $ab = m^3$ is a cube then a and b are themselves cubes.

T F (g) If $\gcd(a,b) = 1$ then $\gcd(a^2, b^2) = 1$

T F (h) There is no primitive Pythagorean triple $x^2 + y^2 = z^2$ with $x = 18$.

T F (i) $6|a^2 \Rightarrow 6|a$.

T F (j) $\{-1, -2, -3, 1, 2, 3\}$ is a complete set of residues mod 6.

9. (8 points) Prove that $\sqrt[3]{\frac{25}{12}}$ is irrational. (Give a proof, don't just state a theorem).

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