Name:

MATH 506 Number Theory – **Exam I**

Wednesday February 15, 2006

Check that that you have all four pages - note that pages are double-sided

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1. (6 points) Find integers q and r such that $-29 = 7q + r$ with $0 \leq r < 7$.

2. (14 points) (a) Use the Euclidean algorithm to compute the greatest common divisor of 1591 and 629.

(b) Write this gcd as a linear combination of 1591 and 629.

3. (8 points) Show that $gcd(4a + 5, 5a + 7) = 1$ or 3 for any integer a. When is it 3?

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4. (8 points) Use the definition of divisibility to prove that if $a \mid b$ and $b \mid c$ then $a \mid 2b + 3c^2$.

5. (20 points) Circle True (T) or False (F).

T F (a) Every integer is a linear combination of 14 and 25.

T F (b) If $a^2|b^3$ then $a|b$.

T F (c) If x is irrational then x^2 is irrational.

T F (d) If $b = aq + r$ then $gcd(a, b) = gcd(a, r)$.

T F (e) If $d \mid a$ and $d \mid b$ then $d \mid [a, b]$.

T F (f) $(2n + 1)n$ is a triangular number for all positive integers n.

T F (g) By the well ordering principle every non-empty set of integers has a least element.

T F (h) For any integer b there exist integers q, r with $b = 8q + r$ and $-8 < r \leq 0$.

T F (i) The smallest positive integer in the set $\{18x + 12y + 8z \mid x, y, z \in \mathbb{Z}\}$ is 2.

T F (j) If $a, b \in \mathbb{N}$ and $d = gcd(a, b)$ then $lcm(a/d, b/d) = ab/d^2$.

6. (8 points) Show $2y^2 - 3z^2 = 1$ has no integer solutions.

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7. (8 points) Let f_n denote the nth Fibonacci number. Prove that $f_2 + f_4 + \cdots + f_{2n} = f_{2n+1} - 1$.

8. (12 points) Decide whether the following linear Diophantine equations have any solutions. If so give the general solution, if not say why there are no solutions. There is no need to go through the Euclidean algorithm to find obvious gcds or solutions.

(a) $8x - 14y = 10, \quad x, y \in \mathbb{Z}$.

(b) $42x + 35y = 17, \quad x, y \in \mathbb{Z}$.

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9. (8 points) Use induction to prove that $7|(3^{2n+1} + 2^{n+2})$ for all positive integers n

10. (8 points) Prove that $\sqrt{2}$ is irrational.

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