Name:

MATH 506 Number Theory – **Exam I**

Wednesday February 25, 2004

Check that that you have all three pages - note that page two is on the back of page one

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1. (7 points) Find integers q and r such that $-73 = 12q + r$ with $0 \leq r < 12$.

2. (12 points) (a) Use the Euclidean algorithm to compute the greatest common divisor of 851 and 222.

(b) Write this gcd as a linear combination of 851 and 222.

3. (7 points) Show that $(3a + 5, 2a + 3) = 1$ for any integer a.

4. (7 points) Give (with justification) a non-trivial factor of $2^{88} + 1$.

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5. (8 points) Use the definition of congruence to prove that if $a \equiv 3 \pmod 4$ then $a^2 \equiv 1 \pmod 8$.

6. (12 points) Decide whether the following linear Diophantine equations have any solutions. If so give the general solution, if not say why there are no solutions. There is no need to go through the Euclidean algorithm to find obvious gcds or solutions.

(a) $15x - 33y = 17, \quad x, y \in \mathbb{Z}$.

(b) $10x - 14y = 8, \quad x, y \in \mathbb{Z}$.

7. (7 points) Use congruences to show that the Diophantine equation has no solutions:

$$x^3 - 7y^3 = 3, \qquad x, y \in \mathbb{Z}.$$

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8. (8 points) Use the definition of divisibility to prove that if $a \mid b$ and $b \mid c$ then $a^2 \mid bc$.

9. (24 points) Circle True (T) or False (F).

T F (a) $(-33, -21) = -3$.

T F (b) If $a \leq b$ then $[a, c] \leq [b, c]$.

T F (c) If $a \mid b$ and $c \mid d$ then $ac \mid bd$.

T F (d) If $a \mid b$ and $c \mid b$ then $ac \mid b$.

T F (e) If $a \mid d$ and $b \mid d$ then $[a, b] \mid d$.

T F (f) Any even integer can be written as a linear combination of 10 and 22.

T F (g) $(15 - 6q, 6) = 3$ for any integer q.

T F (h) For any integer b there exist integers q, r with $b = 7q + r$ and $|r| \leq 3$.

T F (i) The least residue of -176 modulo 5 is -1.

T F (j) $76^{100} \equiv 1 \pmod{7}$.

T F (k) $\{0, 1, 2, -1, -7\}$ is a complete system of residues modulo 5.

T F (l) If $2a \equiv 6 \pmod{10}$ then $a \equiv 3 \pmod{10}$.

10. (8 points) Use congruences to describe the integers n for which $\frac{1}{5}(n^4 + 4)$ is **not** an integer?

PG SCORE/40