Adv. Calc Test 1 2nd Sem Name

1] Let $f: \mathbb{R} \to \mathbb{R}$ be a continuous function on $\mathbb{R}$. State whether the following are true or false. If true, give a brief explanation. If false, give a counterexample. You may refer to the function graphed below for counterexamples.

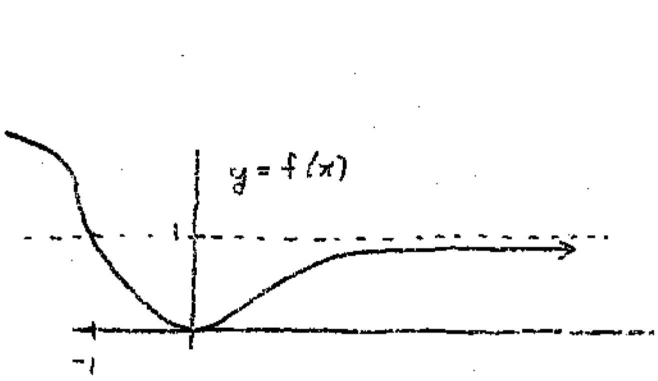
a) f (Open set) = Open set

b) f (Bounded set) = Bounded set

c) f (Closed set) = Closed set

d) f (closed-bounded set) = closed bounded set

e) f ([illegible] interval) = [illegible] interval



Do any <u>4</u> of the following.

2] Show that for any vectors $\vec{u}, \vec{v} \in \mathbb{R}^n$
$|\vec{u}-\vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2\,\vec{u}\cdot\vec{v}$. Draw a diagram illustrating $\vec{u}$, $\vec{v}$ and $\vec{u}-\vec{v}$ and explain what trig. identity this equation represents.

3] Show that if f is a monotone increasing function on an interval $[a,b]$, then f has bounded variation on $[a,b]$ and find V_f, the variation of f on $[a,b]$.

4] Use an ε-δ argument to prove that

$$\lim_{(x,y)\to(0,0)} \frac{x+y}{xy}\sin(xy^3+x^3y) = 0.$$

(where x, y are restricted to domain where $x \neq 0$ and $y \neq 0$)

5] Deduce the Bolzano-Weierstrass Theorem from the Heine-Borel Theorem.

6] Suppose that ~~[illegible]~~ $f: D \to \mathbb{R}$ is a continuous real valued function on a <u>region</u> D in $\mathbb{R}^n$ such that f is positive on the interior of D. Prove that f is nonnegative on all of D.

7] Suppose that $\vec{F}: \mathbb{R}^n \to \mathbb{R}^m$ is a function such that $\vec{F}^{-1}(U)$ is open for every open subset U of $\mathbb{R}^m$. Prove that $\vec{F}$ is continuous on $\mathbb{R}^n$. $\left(\vec{F}^{-1}(U) = \{\vec{p} \in \mathbb{R}^n : \vec{F}(\vec{p}) \in U\}\right)$