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DISCRETE MATHEMATICS
Exam 2
April 9, 2008

The point value of each problem is given in the margin. Total = 80 points.

(8) 1. Give the general solution of the recurrence $h_n = -2h_{n-1} + 4h_{n-2} + 8h_{n-3}\,,\ n \geq 3\,.$

(8) 2. Solve the linear recurrence with initial condition: $h_n = 2h_{n-1} + n - 1\,,\ n \geq 1\,,\ h_0 = 0\,.$

(8) 3. Seven men check in their coats as they enter a restaurant. How many ways can the coats be handed back if

(a) No one receives their own coat back. (Simplify.)

b) Two men receive their own coats back, but no one else does. (No simplification required.)

(8) 4. a) The inclusion-exclusion principle for three sets A, B, C states that

$|A \cup B \cup C| =$

b) Determine the number of positive integers from 1 to 200 that are divisible by 4,5 or 7. (Simplify.)

(8) 5. Write down the binomial expansion of $(1+x)^n$ and then use it to evaluate the sums in parts (b) and (c).

a) $(1+x)^n =$

b) $\sum_{k=0}^{n} \binom{n}{k} =$

c) $\sum_{k=0}^{n} (-1)^k \binom{n}{k} \frac{3^{k+1}}{k+1}$

(4) 6. What is the coefficient of xy^2z^5 in $(x+3y-2z)^8$? (You don't need to simplify.)

(4) 7. Let h_n ($n \geq 1$) denote the number of n letter words that can be formed from the letters a, b, c, d, e such that no two $b's$ are adjacent. Find a recurrence relation, with initial conditions, that h_n satisfies. (Do not try to solve for h_n.)

(8) 8. Let $n \geq 3$ be a fixed positive integer. Determine the number of integer solutions (x, y, z) of the equation

$$x + y + z = n, \quad 3 \leq x \leq 6, \quad y \geq 0, \quad 0 \leq z \leq 12$$

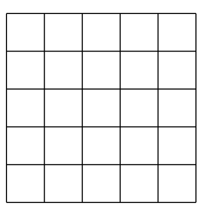
(Leave your answer as a sum/difference of binomial coefficients involving n.)

(8) 9. Let h_n denote the number of ways to choose n pieces of fruit from a choice of apples, bananas, pears if the bananas come in bunches of six, there must be at least three pears and there must be at most five apples. (Assume there is an unlimited supply of each fruit.)

a) Determine the generating function $f(x) = \sum_{k=0}^{\infty} h_n x^n$ and simplify your answer to a rational function in reduced form.

b) Evaluate h_n. Give a simplified answer.

(8) 10. How many ways can 5 nonattacking rooks be placed on a 5-by-5 chessboard with forbidden positions (row,col)=(1,1),(1,2),(2,1),(2,2),(4,5),(5,4),(5,5). Simplify your answer.



(8) 11. Let $\{f_n\}_{n\geq 0}$ be the Fibonacci sequence $f_0 = 0, f_1 = 1, f_2 = 1, \ldots,$ $f_n = f_{n-1} + f_{n-2}$ for $n \geq 2$. Prove by induction that for $n \geq 0$,

$$f_0 + f_2 + f_4 + \cdots + f_{2n} = f_{2n+1} - 1.$$