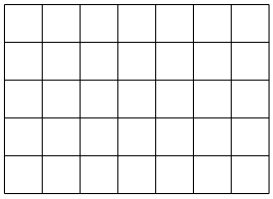
Name _____ Instructor: Todd Cochrane

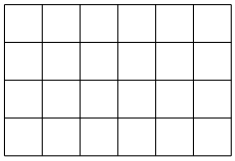
DISCRETE MATHEMATICS
Exam 1
February 20, 2008

The point value of each problem is given in the margin. Total = 80 points.

(6) 1. Three corners of a 5×7 chessboard are removed. Can the remaining 32 squares be perfectly covered with dominoes? Explain.



(6) 2. A 4×6 chessboard is perfectly covered by 12 dominoes. Prove that there must be either a horizontal or vertical fault line (a line that passes through no domino, but not an edge line.)

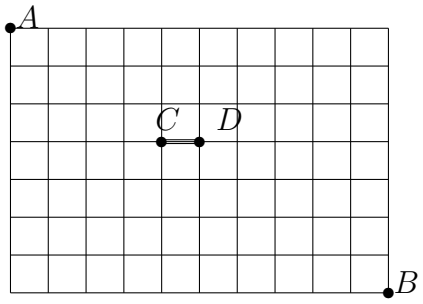


(6) 3. 50 integers are chosen at random from the set $\{1, 2, 3, 4, \ldots, 239, 240\}$. Prove that there are two that differ by at most 4.

(6) 4. A basket holds three kinds of fruit, bananas, apples and pears (at least 20 of each). How many pieces of fruit must be selected in order to guarantee that you have selected at least 4 apples or 5 bananas or 10 pears? Give the minimal number. Justify that your answer suffices by giving a proof by contradiction or quoting an appropriate theorem.

(8) 5. (a) How many paths are there from A to B in the grid below, if you are only allowed to travel right R or down D?

(b) How many paths are there from A to B if you are not permitted to travel on the segment $\overline{CD}$?



(6) 6. Prove that $r(3,3) \leq 6$, that is if the edges of a K_6 (complete graph on 6 vertices) are colored red or blue then there must either be a blue triangle or a red triangle (that is, a blue K_3 or red K_3.)

(12) 7. How many 6 character passwords can be made using only the characters 1,2,3,4,a,b,c,
(a) Assuming that no character is used more than once.

(b) Repeats are allowed as long as they are adjacent.

(c) Repeats are allowed but exactly two of the characters must be letters.

(6) 8. Determine the number of full houses in a poker hand. (5 cards with 3 of one rank and 2 of a different rank. Recall there are 13 ranks and 4 distinct cards of each rank.)

(6) 9. Count the number of permutations of the letters AABBBCCCC.

(6) 10. Ten people are seated around a circular table, 4 men and 6 women. How many circular arrangements are there if Bob (one of the 4 men) refuses to sit next to another man.

(6) 11. How many ways can 6 nonattacking rooks be placed on an 8×8 chessboard if 1 is blue, 2 are red and 3 are green. (They are indistinguishable other than the color, and any two rooks in the same row or column may attack one another.)

(6) 12. Count the number of integer solutions to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 20,$$

with $x_1 \geq 3\,,\ x_2 \geq -2\,,\ x_3 \geq 0\,,\ x_4 \geq 0\,,\ x_5 \geq 0\,,\ x_6 \geq 0\,.$